

Anderson localization in a string of microwave cavities

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The field distributions and eigenfrequencies of a microwave resonator which is composed of 20 identical cells have been measured. With external screws the periodicity of the cavity can be perturbed arbitrarily. If the perturbation is increased a transition from extended to localized field distributions is observed. For very large perturbations the field distributions show signatures of Anderson localization, while for smaller perturbations the field distribution is extended or weakly localized. The localization length of a strongly localized field distribution can be varied by adjusting the penetration depth of the screws. Shifts in the frequency spectrum of the resonator provide further evidence for Anderson localization. [S1063-651X(99)10310-6]

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I. INTRODUCTION

In 1958 Anderson calculated the effects of perturbations of a periodic lattice on the eigenvalues and eigenfunctions of the Schrödinger equation [1]. He was able to show that for a variety of disordered potentials the eigenfunctions are exponentially localized in a small region of the lattice due to the interference of waves scattered from the perturbations or impurities. Until then the localization of waves played a crucial role in almost every physical domain—for a review see, e.g., [2]. In the case of a long, disordered one-dimensional (1D) chain it could be shown rigorously, that Anderson localization occurs for a very large class of potentials [3–5]. However there are classes of disordered potentials where extended states do exist [6,7]. It has been conjectured that localization effects are far more general [8–10] and are a generic feature of wave equations [11,12].

Experiments working with electromagnetic waves are usually focusing on secondary features of localization, e.g., coherent backscattering [13,14] or the investigation of the transmission and absorption coefficients [15–17]. A direct search for localization in electromagnetic field distributions has only very recently been attempted in two-dimensional (2D) [18] and three-dimensional (3D) systems composed of 1D waveguides [19]. In the present paper we report on the direct observation of Anderson localization in an electromagnetic system, i.e., a microwave resonator, governed by the *vectorial* Helmholtz equation, with a perturbed periodicity. So far microwave resonators have been a major experimental tool for the investigation of so-called quantum billiard systems [20–26], i.e., a pointlike particle caught in a potential with infinitely high walls, or recently for the study of models used in nuclear physics [27]. In all these experiments the analogy between the *scalar* Helmholtz equation which describes the electromagnetic field inside a *flat* microwave cavity and the Schrödinger equation is used and the energy spectra of the resonators are statistically analyzed—for a review of a wide range of experiments and the statistical methods used see, e.g., [28].

Here we will discuss experiments performed with a so-called three-dimensional microwave cavity, i.e., a resonator

that has to be described by the vectorial Helmholtz equation and belongs to the class of resonators investigated in [29–32]. As mentioned earlier these experiments compose a direct experimental search for Anderson localization in such systems, not to be confused with the study of higher dimensional lattices described by the Schrödinger equation [33]. One of the main problems in such an experimental investigation is the limited size of the system at hand. In numerical simulations of finite chains described by the Schrödinger equation the number of elementary cells is usually some hundred cells or even higher (for an early review see, e.g., [34]), while a realistic experimental setup with a chain of microwave resonators has to have a much smaller number of elementary cells to keep the dimension of the system at an acceptable level.

The paper is organized as follows. In Sec. II we will give a description of the microwave resonator and the experimental methods used to measure the resonance frequencies and the field distributions of the cavity. After having verified that the unperturbed system is indeed periodic and that we find extended states we proceeded to investigate the influence of a single perturbation of the periodicity on the eigenstates and eigenfrequencies—the results are given in Sec. III. As a third possible configuration we tried to setup a disordered chain and measured the field distributions discussed in Sec. IV.

II. EXPERIMENT

To simulate periodic and perturbed systems, an accelerating cavity of the superconducting Darmstadt linear electron accelerator S-DALINAC [35] has been modified. The cavity itself is manufactured from 2-mm-thick Niobium sheetmetal and consists of 18 identical cells and two slightly different cells at the ends of the chain to compensate the influence of the cutoff tubes attached to the outer cells. The cylindrical symmetric section with the 20 cells has a length of 1 m and the diameter of a single cell varies between 91 mm and 39 mm. A sketch of the modified accelerating cavity is given in Fig. 1. Below 3.5 GHz the two cutoff tubes cause an exponential decay of the electromagnetic field outside the 20 cell

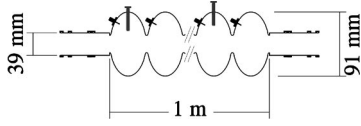


FIG. 1. Sketch of the two end sections of the modified accelerating cavity. Besides the identical dipole antennas two of the adjustable lead screws are shown.

section [36] and above 3.5 GHz two niobium plates can be used to close the first and the last cell. In both cases only the 20 cell section is excited up to 20 GHz by a HP8510B vectorial network analyzer connected to a set of 20 identical and periodically mounted capacitively coupling dipole antennas. We chose a large penetration depth of 8 mm for the antennas to ensure that even modes which are localized in just one or two cells can be observed in the transmission spectra. Care has been taken to ensure that all antennas are identical and periodically mounted so that the periodicity of the resonator is not perturbed by the antennas themselves. To perturb the periodic setup every second cell is equipped with an adjustable lead screw with a diameter of 20 mm which can penetrate into the resonator's volume. The penetration depth of each screw $d^i = 0$ mm, $i = 1, 3, \dots, 19$ can be continuously varied between 0 and 47 mm. The cavity therefore allows the investigation of three different setups, i.e., a periodic system, a system with a single perturbation or impurity with variable strength and a disordered system where the d^i 's are set to random values. Despite the fact that the measurement of the field distributions has been performed at room temperature, all materials used will become superconducting at temperatures which can easily be reached inside a LHe-bath cryostat described in [28] to allow future high resolution measurements of the cavity's spectra.

The electromagnetic fields \vec{E} and \vec{B} inside the cavity are described by the vectorial Helmholtz equation [36]

$$\left(\Delta + \epsilon\mu \frac{\omega^2}{c_0^2} \right) \vec{E}(\vec{r}) = \vec{0} \quad (1)$$

and

$$\left(\Delta + \epsilon\mu \frac{\omega^2}{c_0^2} \right) \vec{B}(\vec{r}) = \vec{0} \quad (2)$$

with the corresponding boundary conditions

$$\vec{E}_{\parallel}(\vec{r})|_{\partial G} = \vec{0} \quad \text{and} \quad (3)$$

$$\vec{B}_{\perp}(\vec{r})|_{\partial G} = \vec{0}$$

on the walls δG which are assumed to be ideally conducting. The well known analogy between flat electromagnetic cavities and two-dimensional quantum potentials, that has stimulated a multitude of experiments [28] is, of course, lost. As mentioned above the effects of Anderson localization can be found either in the eigenfunctions, in our case the electromagnetic field distributions, or the energy spectrum, in our case the set of resonance frequencies, of a system. We examined therefore both, the field distributions and the energy spectrum, for signatures of localization effects.

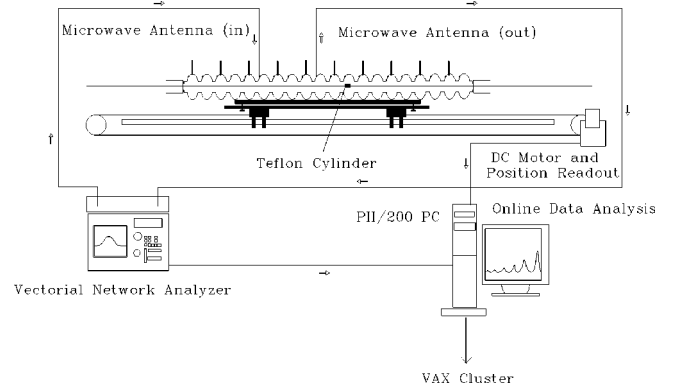


FIG. 2. Sketch of the experimental setup we used to measure the field distributions of the resonator. The cavity is moved by a DC motor on a sledge while the Teflon cylinder is fixed on the axis. A Pentium-133 PC is used for an online data analysis.

The electromagnetic field amplitudes inside the cavity can be measured using Slater's theorem formulated in [37] that describes shifts of the resonance frequencies of a microwave cavity if a perturbing body is brought into the resonating volume. The electric and the magnetic field energies inside a microwave cavity which is excited in resonance are of equal magnitude if the cavity is in a stationary state. If an external body is introduced into the resonator, the field energies are shifted and the resonance frequency f_0 will be adjusted in a way that the electric and magnetic field energies become equal again and a new stationary state is reached. According to [37] the frequency shift $\Delta f = f - f_0$ of a resonance caused by a perturbing body of volume ΔV is

$$\frac{\Delta f}{f_0} = \frac{1}{4U} \int_{\Delta V} (\epsilon\epsilon_0 \vec{E}^2 - \mu\mu_0 \vec{H}^2) dV, \quad (4)$$

where U is the total energy stored in the electromagnetic field and ϵ and μ are the permittivity and the permeability of the perturbing body. If ΔV is small compared to the wavelength of the mode under investigation, the perturbing body is moved on a nodal line of the magnetic field (i.e., $\vec{H} = \vec{0}$) and is composed of a dielectric material, Eq. (4) can be written as

$$\frac{\Delta f}{f_0} = \frac{\epsilon\epsilon_0 \Delta V}{4U} \vec{E}^2, \quad (5)$$

which immediately leads to the proportional relation

$$\Delta f \propto \vec{E}^2. \quad (6)$$

The squared field strength \vec{E}^2 can therefore be calculated directly from measurements of the frequency shift caused by a small perturbing body.

To measure the frequency shift we used the experimental set-up sketched in Fig. 2. The resonator is moved with a speed of 1 m/min while the perturbing body is fixed on the symmetry axis of the cavity with a very thin string, to prevent oscillations of the body. The first 20 resonances of the cavity are transverse magnetic (TM) modes for which the magnetic field is zero on the axis. We further used a cylindrical Teflon bead as a perturbing body with a volume of

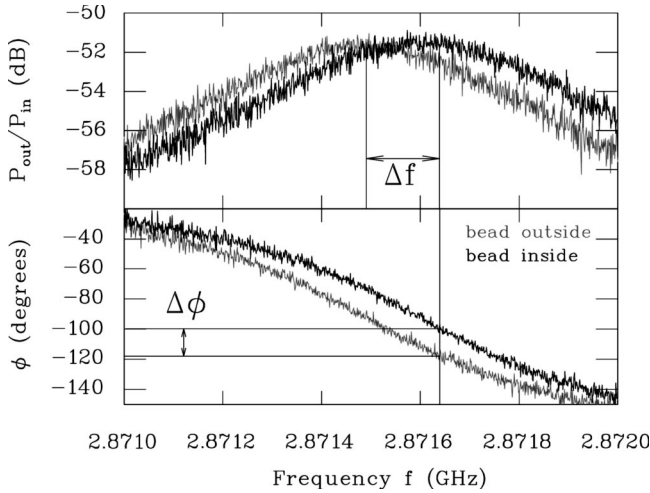


FIG. 3. Transmission spectra (top) and phase relations ϕ between the received and the emitted wave (bottom), showing the perturbation introduced by the Teflon cylinder.

$\Delta V = 2 \text{ mm}^3$ and a permittivity of $\epsilon \approx 2.1$ and a permeability of $\mu \approx 1$ [38]. The special symmetry of our system and the perturbing body we are using allows us to measure \vec{E}^2 directly by using Eq. (6) unlike similar experiments [32], where a metallic perturbing body is used and the composed quantity $-2\vec{E} + \vec{B}$ is measured.

The network analyzer can be set to continuously sweep a range of 1 MHz across the last position of the resonance which takes a time of 184 ms. In the upper part of Fig. 3 a part of the transmission spectrum with a resonance at 2.8715 GHz and the Teflon cylinder outside and inside the resonator is shown. The frequency shift Δf caused by the perturbation of the electromagnetic field is clearly visible. Nevertheless, additional noise blurs the position of the resonance so that it is impossible to find its position by simply looking for the frequency with the highest transmission. Transmitting the data via the IEEE bus for an offline analysis with, e.g., fitted resonance curves is also not practical since during the time needed for the data transfer ($\approx 1 \text{ s}$) the cavity moves about 16 mm. Beside the measurement of the ratio between the received and the emitted wave, the HP8510B allows the analysis of the phase relation ϕ between the received and the emitted wave which shifts by π at the resonance frequency. For a resonance with a frequency f_0 and a quality factor Q one can write the phase relation as [36]

$$\phi = \arctan\left(\frac{ff_0}{Q(f_0^2 - f^2)}\right) + \pi/2 \quad (7)$$

which can be expanded around the point $f = f_0$ to a linear relation

$$\phi = \frac{\pi}{2} + 2 \frac{Q(f_0 - f)}{f_0} + \mathcal{O}(f^2). \quad (8)$$

As one can see in the lower part of Fig. 3 the frequency shift Δf is small compared to the region where the linear series approximation of Eq. (7) is applicable. The network analyzer is therefore set to emit a wave with the frequency f_0 and measure the phase relation ϕ between the received and

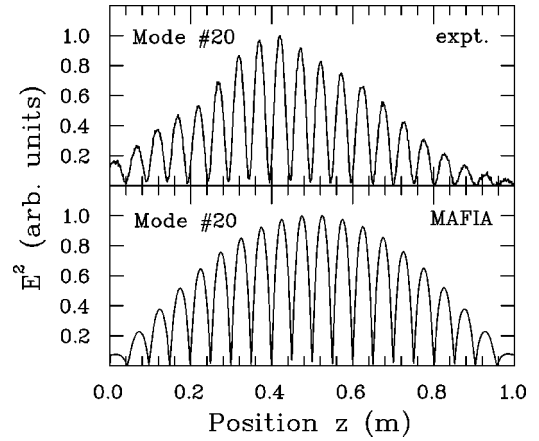


FIG. 4. Field distributions $\vec{E}^2(z)$ of the unperturbed cavity, measured with our setup (expt.) and calculated with the finite element code MAFIA. The position on the axis of the cavity is denoted by z .

the emitted wave. Since no time consuming sweep is required for measurements performed at a constant frequency, the Teflon bead is assumed to be fixed during each measurement and the received signal can be averaged up to 4000 times, therefore greatly reducing the underlying noise. The difference of the phase relations with the bead outside and inside the cavity, $\Delta\phi$, as shown in the lower part of Fig. 3, can then easily be used with Eq. (8) to compute the field amplitudes. From Eqs. (6) and (8) one gets the proportionality relation

$$\Delta\phi \propto \Delta f \propto \vec{E}^2. \quad (9)$$

In a first step we measured the field distribution of the unperturbed cavity, i.e., the penetration depths of the screws were set to zero, and compared our measurements with finite element calculations using the MAFIA [39] computer code. The model used for the finite element calculations is cylindrically symmetric with a resolution of $1 \text{ mm} \times 0.25 \text{ mm}$ and allows the calculation of the resonance frequencies and the field distributions of the first 40 TM modes. In Fig. 4 the experimental and the calculated electric field distributions for the 20th mode are compared, showing the effects of small deviations from the ideal geometry, either due to the additional holes that were drilled into the cavity or the cut-off tubes, are negligible.

III. A SINGLE PERTURBATION

If the system does show a transition from extended to localized states if the periodicity is perturbed—despite the finite size and the fact that it has to be described by the vectorial Helmholtz equation—one expects for a single perturbation of the otherwise periodic chain only *one* local impurity mode in each band with a localized field distribution. We therefore investigated the development of the field distribution of the first resonance if the penetration depth of an arbitrary screw (here the one in cell no. 13) is increased. The evolution of the wave function is shown in Fig. 5, where we plotted the frequency shift Δf against the position z of the Teflon bead. Beside perturbing the periodicity of the cavity

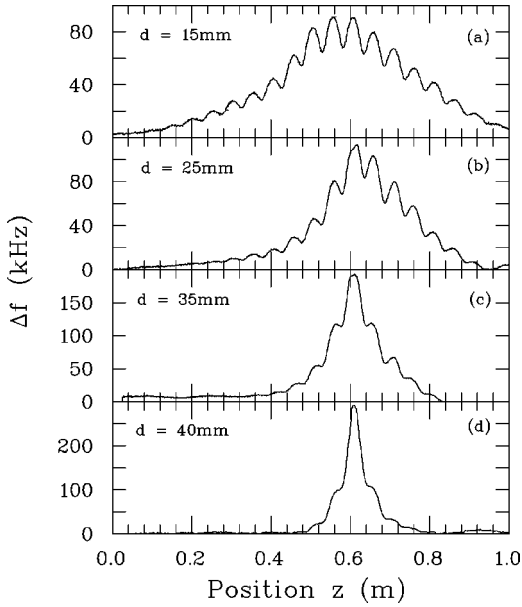


FIG. 5. Shift Δf in the resonance frequency of the first resonance caused by the Teflon cylinder for different penetration depths d^{13} of the screw in cell no. 13.

the screw also breaks its cylindrical symmetry so that the cavities axis is not a nodal line of the magnetic field anymore. Calculations with MAFIA have nevertheless shown that the magnetic field on the axis is still zero for a wide range of penetration depths. For penetration depths above approximately 40 mm the actual electric field is slightly larger than the one determined by Eq. (9) since the proportionality relation has to be written as

$$\Delta\phi \propto \Delta f \propto \epsilon \epsilon_0 \vec{E}^2 - \mu \mu_0 \vec{H}^2. \quad (10)$$

For $d^{13} = 15$ mm the first resonance excites the whole cavity [Fig. 5(a)] while for an increasing d^{13} the field amplitude drops down to zero almost everywhere in the resonator. The field distribution of the first resonance even localizes up to a point where *only* the disturbed and a few neighboring cells are excited [Fig. 5(d)]. As it is the case for systems described by the Schrödinger equation, the field distributions of the other 19 modes of the first TM band are still extended over the whole cavity.

In the case of strong localization in quantum systems the envelope of the localized eigenfunction can for a wide range of perturbed potentials be approximated by an exponentially decaying function [1,3], i.e.,

$$|\psi_{loc}(z)|^2 \propto \exp\left(-\frac{|z-z_0|}{L_{loc}}\right), \quad (11)$$

where z_0 is the position of the perturbation and L_{loc} is called the localization length.

Despite the fact, that even for strongly localized modes the field distribution is not symmetric with respect to z_0 —mainly because of the small dimensions of our system—Eq. (11) can be used to describe *the envelope* of \vec{E}^2 for a wide range of penetration depths d^{13} . In Fig. 6 one can see a nearly linear dependence between d^{13} and L_{loc} for penetration depths above 35 mm. For a smaller perturbation the field

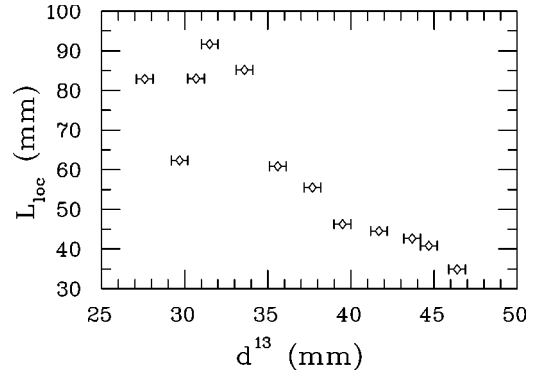


FIG. 6. Localization length L_{loc} , i.e. the length where \vec{E}^2 decreases by $1/e$, for various penetration depths d^{13} . For penetration depths smaller than 35 mm the field distributions are only weakly localized and L_{loc} is not well defined.

distribution shows a clear maximum [see, e.g., Fig. 5(b)] around the perturbing screw, but Eq. (11) cannot be used to describe the envelope of \vec{E}^2 , since the decay is not exponential, and L_{loc} is therefore not well defined. To underline the point that the field distribution has a maximum in the disturbed cell one usually calls this form of localization *weak localization* [40]. The behavior of the field distributions of our microwave resonator can be compared to the eigenfunctions computed for numerical models with a single perturbation of a periodic chain [34].

Beside the transition from an extended to an exponentially localized eigenfunction the solutions of the Schrödinger equation do have other prominent features in periodic potentials with a single perturbation. One of these is a shift in the eigenvalue of the local impurity mode, while the eigenvalues of the solutions in a band that are still extended over the system remain fixed. An investigation of the resonance frequency f_0 of the first eight modes of the microwave resonator while d^{13} is increased, see Fig. 7, shows that only the local impurity mode—in this case the lowest excitation of the cavity—experiences a notable shift in its resonance frequency. Even in the regime of weak localization, i.e., around $d^{13} \approx 30$ mm, where the field distributions of all modes are extended over the whole cavity, no shift of the resonance frequency of the first mode is observed. The transition from an extended or weakly localized state to a strong, i.e., expo-

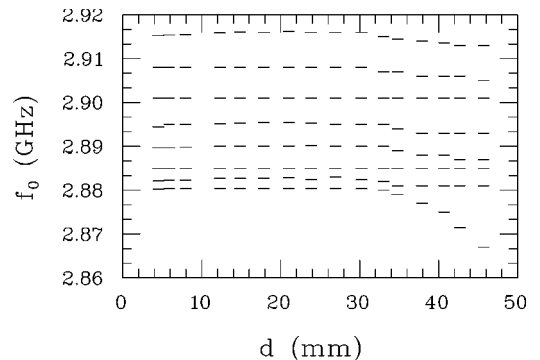


FIG. 7. Resonance frequencies f_0 of the first eight modes for different penetration depths d^{13} . A notable frequency shift which is going along with a localization of the field distribution can only be observed for the first mode.

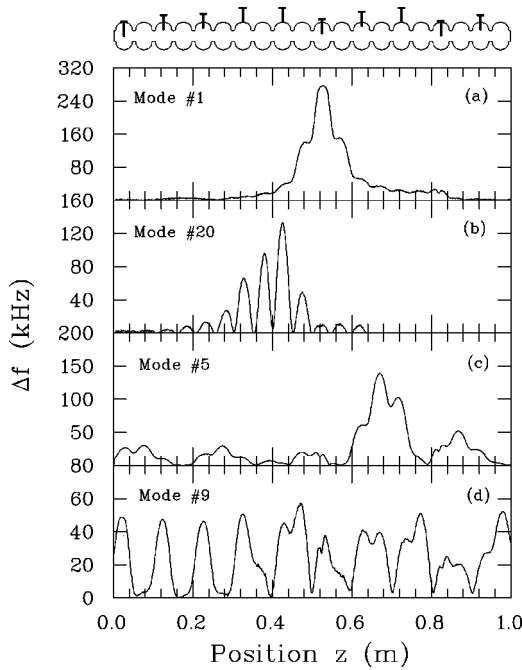


FIG. 8. Field distributions of a disordered cavity—the positions of the screws are sketched above the plot. While on the edges of the first band the modes are strongly localized [(a) and (b)] there are still weakly localized modes (c) or even extended modes (d) in the middle of the band.

nential, localized state can therefore not only be observed by measuring the eigenfunctions but also by looking at the eigenfrequencies. To conclude this section we like to state that we found a complete correspondence between the eigenfunctions and eigenvalues of the Schrödinger equation in a periodic potential with a single perturbation and the field distributions and resonance frequencies of a periodic 3D microwave cavity with a single, sufficiently large perturbation.

IV. MULTIPLE PERTURBATIONS

In contrast to the emerging of a local impurity mode at the position of a single perturbation or impurity, one expects for a large class of one-dimensional disordered systems that *all* the eigenfunctions of a scalar wave equation are exponentially localized [1,3–5]. For disordered acoustic systems a coexistence between extended, weakly localized and strongly localized modes has been reported by He and Maynard [41]. To examine the behavior of a disordered electromagnetic system we set the penetration depths of all screws to various, non periodic alternating lengths. The positions of the screws are sketched in an inset above Fig. 8 where also several different field distributions are shown.

In contrast to the predictions for one-dimensional systems in [1,3–5] not all field distributions are exponentially localized as for example the first [Fig. 8(a)] or the 20th [Fig. 8(b)] mode with localization lengths of $L_{loc}^1 \approx 45$ mm and $L_{loc}^{20} \approx 60$ mm. We still found states which show weak localization, e.g., the fifth mode [Fig. 8(c)], or that are still extended throughout the whole system [ninth mode, Fig. 8(d)]. Dean and Bacon found a similar behavior in an early numerical model where they studied the eigenfunctions of a disordered harmonic chain composed of 22 light atoms and 28 heavy

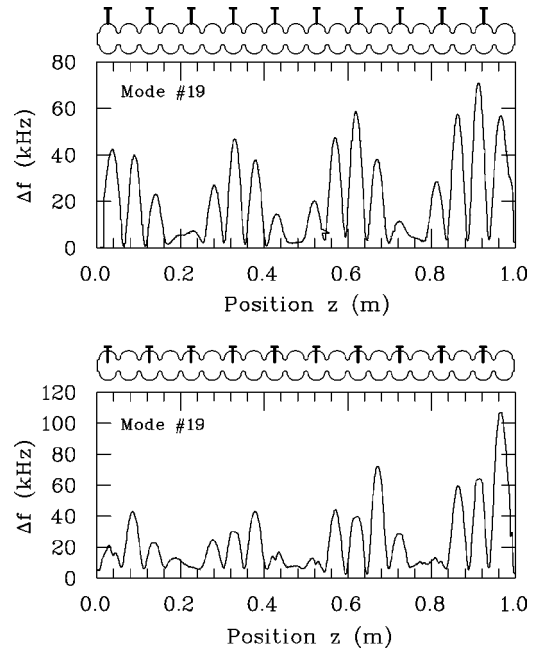


FIG. 9. Field distribution of the 19th mode in the unperturbed cavity (top) and in the *periodically* perturbed cavity (bottom). Both states are extended over the whole resonator and have similar envelopes.

ones and which is described by the Schrödinger equation [42]. In their model, localization occurs always on the upper edge of a band, while we are observing localization at both, the upper and the lower edge of the first band. The extended modes we are observing are found in the middle of the band like the ninth mode shown in Fig. 8(d). As mentioned above a coexistence of localized and extended modes has been already observed for an acoustic system [41].

Our particular setup allows the examination of another interesting system: If the penetration depths off all screws are set to the same value, the system will be periodic again—an elementary cell is now composed of an unperturbed and a perturbed cell—and should therefore show only extended wave functions. As an example we compare in Fig. 9 the field distributions of the 19th mode of the cavity with $d^i = 0$ mm, $i = 1, 3, \dots, 19$ and $d^i = 40$ mm, $i = 1, 3, \dots, 19$. As expected we found, that both wave functions have essentially the same envelope, despite the fact that for a *randomly* perturbed setting the 19th mode is strongly localized. In both cases the field distribution is not exactly symmetric—an effect that is caused by geometrical imperfections and which is also visible in the upper part of Fig. 4.

V. CONCLUSION

By using an appropriately shaped 3D microwave cavity we investigated a finite, periodic or disordered system with eigenfunctions described by the vectorial Helmholtz equation. The field distributions inside the cavity were measured by analyzing the phase shifts caused by a small dielectric body inside the resonator. In the case of a chain of quasi-identical cells we find extended field distributions—the whole resonator volume is excited.

It is well known that the eigenstates of a scalar wave equation are exponentially localized in the case of infinitely

long, disordered chains and that there is always one local impurity mode per band in the case of a single perturbation or impurity of the chain. Our experiments showed that the same behavior can be observed for a finite—in fact with just 20 cells very small—system although it is described by the vectorial Helmholtz equation. We observed the transition from an extended state to an exponentially localized state in the case of a single perturbation by looking at the field distribution and the resonance frequency of the mode. In the regime of strong localization we found a linear dependence between the localization length and the penetration depth of the screw that causes the perturbation. The current setup does not allow the investigation of totally disordered chains but enables us to study systems where extended modes coexist with strong and weakly localized ones which can be compared to certain numerical models for scalar problems [42] or results found in acoustic systems [41]. It is also possible to study chains composed of a periodic array of two-cell elements in which *all* states are extended over the whole cavity.

Our experiments showed that appropriately shaped microwave cavities exhibit all the features of extended periodic systems—despite the fact that they are finite and in our case

have to be described by vectorial wave equations. The localization of an eigenfunctions of the vectorial Helmholtz equation can be observed in this electromagnetic system. The experimental verification of predictions on the statistical behavior of extended billiard chains, see, e.g., [43], should therefore be possible and combine the results of the theory of periodic systems and so-called quantum chaotic systems [28]. The experimental setup itself already allows the study of various problems from the fields of solid-state physics and scattering problems in a very clean and pedagogical way.

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